Market Coverage and the Existence of Equilibrium in a Vertically Differentiated Duopoly*

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Abstract

The existence of a pure-strategy subgame perfect equilibrium in qualities and prices is investigated in a duopoly model of vertical differentiation where quality improvements require a quadratic variable cost. The alternative cases of partial and full market coverage are considered. It is shown that there exists a parameter range where the incentive to decrease differentiation arises for the high-quality firm, preventing firms to reach a pure-strategy duopoly equilibrium.

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1 Introduction

In the existing literature on vertical product differentiation, quality improvements imply alternatively a fixed or a variable cost. The nature of technology largely affects the equilibrium market structure (for a review, see Anderson et al., 1992, chapter 8). The well known finiteness property obtains when quality improvements require either a fixed cost possibly represented by R&D efforts, or a variable cost which does not increase too fast as quality increases (Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983). Otherwise, with sufficiently convex variable costs of quality, a segmented market structure obtains, as in horizontal differentiation models à la Hotelling (1929). As stressed by Gabszewicz and Thisse (1986), vertical differentiation models are generally expected to generate pure-strategy equilibria where prices are strictly above marginal production costs. On the contrary, under horizontal product differentiation, an established result is that a pure-strategy equilibrium in prices may not always exist (see, inter alia, d'Aspremont et al., 1979; Gabszewicz and Thisse, 1986; Economides, 1986; Anderson, 1988). More precisely, a subgame perfect equilibrium with prices greater than marginal cost may fail to exist, because firms' location choices drive prices to marginal cost.[1]

To our knowledge, all existing contributions on vertical product differentiation assume either partial or full market coverage. The only paper where the extent of market coverage is endogenously determined by firms' strategic interaction is due to Wauthy (1996), analysing a vertically differentiated duopoly where firms produce at no cost. He identifies the parameter ranges where either full or partial market coverage arises at equilibrium, as well as a range where a corner solution at the price stage obtains, in which the low-quality firm's price extracts all the surplus from the individual located at the lower bound of the support of consumer's distribution. He proves that such a corner solution is indeed the pure-strategy subgame perfect price equilibrium in the relevant range.

We consider a duopoly model of vertical differentiation with quadratic costs of quality improvements, so that the finiteness property does not hold.

[1] Obviously, a price equilibrium in mixed strategies always exists (Dasgupta and Maskin, 1986; Osborne and Pitchick, 1987).

We investigate the existence and characterization of pure-strategy subgame perfect equilibria for a fixed market size. The alternative cases of full and partial market coverage are considered. We show that the parameter intervals in which the two alternative regimes can arise are disjoint. In order to define the demand structure in the parameter range where neither partial nor full market coverage can be properly defined, we prove that the low-quality firm sets her price to extract all the surplus of the poorest consumer in the market. Our findings reveal that, in such interval of demand parameters, a pure-strategy equilibrium fails to exist. This is due to the incentive for the high-quality firm to set a quality such that the rival's sales are driven to zero. Should firms produce at zero cost, as in Wauthy (1996), the non-existence problem would disappear, due to the incentive for the high-quality producer to supply the highest quality which is technologically feasible.

The remainder of the paper is structured as follows. The model is laid out in section 2, describing the alternative cases of partial and full market coverage. Section 3 contains the proof of the non-existence of a pure-strategy equilibrium with prices above marginal costs. Concluding remarks are presented in section 4.

2 The model

We describe a model of vertically differentiated duopoly under complete information. Each firm produces a vertically differentiated good, with $q_H \geq q_L$, and then competes in prices against the rival. There exists a continuum of consumers indexed by their marginal willingness to pay for quality $\theta \in [\theta_0; \theta_1]$; with $\theta_0 = \theta_1 - 1$: The distribution of consumers is uniform, with density $f(\theta) = 1$, so that the total mass of consumers is also 1. Each consumer buys one unit of the product i that yields the highest net surplus $U = \theta q_i - p_i$; $i = H; L$:

Production technology involves variable costs, which are quadratic in the quality level and linear in the output level:

$$C_i = q_i^2 x_i \quad i = H; L; \tag{1}$$

where x_i indicates the output level of firm i. Firm i's profit function is

$$\pi_i = (p_i - q_i^2) x_i : \tag{2}$$

Competition between firms is fully noncooperative and takes place in two stages. In the first, firms set their respective quality levels; then, in the second, which is the proper market stage, they compete in prices. The solution concept applied is the subgame perfect equilibrium by backward induction. In the remainder of the section, we describe the two alternative equilibria that can arise under either full or partial market coverage.

2.1 Full market coverage

This setting follows the analysis presented in several contributions (Moorthy, 1988; Champsaur and Rochet, 1989; Cremer and Thisse, 1994; Lambertini, 1996). Suppose all consumers are able to buy, i.e., θ_1 is sufficiently high to allow for full market coverage. Given generic prices and qualities, the "location" of the consumer indifferent between the two varieties is $h = (p_H - p_L)/(q_H - q_L)$, so that market demands are $x_H = \theta_1 - h$ and $x_L = h - (\theta_1 - 1)$.

Consider the market stage. From the first order conditions (FOCs henceforth),

$$\frac{\partial \pi_H}{\partial p_H} = \theta_1 - \frac{2p_H - p_L + q_H^2}{q_H - q_L} = 0; \tag{3}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H - 2p_L + q_L^2}{q_L(q_H - q_L)} - (\theta_1 - 1) = 0; \tag{4}$$

the following equilibrium prices obtain:

$$p_H = \frac{(q_H - q_L)(\theta_1 + 1) + 2q_H^2 + q_L^2}{3}; \quad p_L = \frac{(q_H - q_L)(2 - \theta_1) + 2q_L^2 + q_H^2}{3} \tag{5}$$

Substituting and rearranging, we get the profit functions defined exclusively in terms of qualities, $\pi_i(q_H, q_L)$. The subgame perfect quality levels are

$$q_H = \frac{4\theta_1 + 1}{8}; \quad q_L = \frac{4\theta_1 - 5}{8}; \tag{6}$$

which entails the general constraint $\theta_1 \geq 9/4$, in order for the poorest consumer to be in a position to buy the low-quality product. The corresponding equilibrium profits are $\pi_H = \pi_L = 3/16$, and equilibrium demands are $x_H = x_L = 1/2$. Observe that the socially optimal qualities would be the first and third quartiles of the interval $[\theta_0/2, \theta_1/2]$, which obtains from the calculation of the preferred varieties for the richest and the poorest consumer in the market, if such varieties were sold at marginal cost. This implies that (i) qualities are set, respectively, too low and too high as compared to the social optimum; and (ii) this model shares its general features with the model of spatial competition with quadratic transportation costs (see Cremer and Thisse, 1994).[2]

2.2 Partial market coverage

Consider now the case where the market is partially covered. We retain the set of assumptions introduced above, except that now there exists a consumer who is indifferent between buying the low-quality good and not buying at all. His location along the spectrum of the marginal willingness to pay is given by the ratio $k = p_L/q_L$, so that now market demands are $x_H = \theta_1 - h$ and $x_L = h - k$. Given the cost function (1), the profit function of firm i remains defined as in (2).

Again, proceeding backwards, the FOCs for noncooperative profit maximization are

$$\frac{\partial \pi_H}{\partial p_H} = \theta_1 - \frac{2p_H - p_L + q_H^2}{q_H - q_L} = 0, \tag{7}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H q_L - 2p_L q_H + q_H q_L^2}{q_L(q_H - q_L)} = 0. \tag{8}$$

Observe that (7) coincides with (3) since the demand function for the high-quality good is the same in both settings. Solving the system (7-8), one obtains the following equilibrium prices:

$$p_H = \frac{q_H(2\theta_1 q_H + 2q_H^2 - 2\theta_1 q_L + q_L^2)}{4q_H - q_L}; \quad p_L = \frac{q_L(\theta_1 q_H + q_H^2 - \theta_1 q_L + 2q_H q_L)}{4q_H - q_L} \tag{9}$$

as the equilibrium prices. Substituting and simplifying, we get the following expressions defining the firms' profit functions at the quality stage:

[2] It can be shown that, under full market coverage, the spatial model with quadratic transportation costs is actually a special case of a vertical differentiation model with quadratic costs of quality improvement (Cremer and Thisse, 1991).

$$\pi_H = \frac{q_H^2(q_H - q_L)(2\theta_1 - 2q_H - q_L)^2}{(4q_H - q_L)^2}; \quad \pi_L = \frac{q_H q_L(q_H - q_L)(\theta_1 + q_H - q_L)^2}{(4q_H - q_L)^2}. \tag{10}$$

The corresponding FOCs are:

$$\frac{\partial \pi_H}{\partial q_H} = \frac{q_H}{(4q_H - q_L)^3}(16\theta_1^2 q_H^2 - 64\theta_1 q_H^3 + 48q_H^4 - 12\theta_1^2 q_H q_L + 48\theta_1 q_H^2 q_L -$$

$$- 20q_H^3 q_L + 8\theta_1^2 q_L^2 - 12\theta_1 q_H q_L^2 - 12q_H^2 q_L^2 - 8\theta_1 q_L^3 + 9q_H q_L^3 + 2q_L^4) = 0; \tag{11}$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{q_H}{(4q_H - q_L)^3}(4\theta_1^2 q_H^2 + 8\theta_1 q_H^3 + 4q_H^4 - 7\theta_1^2 q_H q_L - 30\theta_1 q_H^2 q_L - 23q_H^3 q_L +$$

$$+ 24\theta_1 q_H q_L^2 + 36q_H^2 q_L^2 - 2\theta_1 q_L^3 - 19q_H q_L^3 + 2q_L^4) = 0; \tag{12}$$

whose solution gives the unregulated Nash equilibrium qualities, $q_H^* = 0.40976\theta_1$ and $q_L^* = 0.199361\theta_1$.[3] Equilibrium prices are $p_H^* = 0.2267\theta_1^2$; $p_L^* = 0.075\theta_1^2$, outputs are $x_H^* = 0.2792\theta_1$; $x_L^* = 0.3445\theta_1$; while profits amount to $\pi_H^* = 0.0164\theta_1^3$; $\pi_L^* = 0.0121\theta_1^3$.

The equilibrium values of firms' profits are acceptable if total equilibrium demand is at most equal to one, i.e., $k \geq \theta_0$; which implies the constraint $\theta_1 \leq 1.6032$. Otherwise, the marginal willingness to pay of the consumer supposedly indifferent between buying the low-quality good and not buying at all falls below the lower bound of the interval assumed for θ. If this is the case, i.e., $\theta_1 > 1.6032$; then the above specification of demand functions is no longer valid.

The discussion carried out in this section leads to the following:

Proposition 1 If $\theta_1 \in [1; 1.6032]$; there exists a unique subgame perfect equilibrium in pure strategies, yielding partial market coverage. If $\theta_1 \geq 9/4$; there exists a unique subgame perfect equilibrium in pure strategies, yielding full market coverage.

In the next section, we investigate the existence of a subgame perfect equilibrium in the interval $\theta_1 \in (1.6032; 9/4)$.

[3] This can be verified through numerical calculations, initially performed by normalizing θ_1 to 1. Then, increasing θ_1 shows that the relationship between equilibrium qualities and θ_1 is linear. The proof that leapfrogging is not profitable is omitted since it is in Motta (1993).

3 A non-existence proof

Suppose $\theta_1 \in (1.6032, 9/4)$. In this interval, the price behaviour of the low-quality firm is described by the following lemma:

Lemma 1 For all $\theta_1 \in (1.6032, 9/4)$, the only candidate as a pure-strategy equilibrium price for the low-quality good is $p_L = (\theta_1 - 1)q_L$.

Proof. Suppose, first, that the price of the low-quality firm is as in the partial coverage case, given by (9). If so, the consumer indifferent between buying the low quality and not buying at all, is located below θ_0. This contradicts the hypothesis of partial market coverage. Suppose now, alternatively, that the price of the low-quality firm is as in (5). In this case, the consumer indifferent between buying the low quality and not buying at all, is located above θ_0, contradicting the hypothesis of full market coverage under which (5) has been obtained. ■

The price set by the high-quality firm is the best reply to p_L:

$$p_H = \frac{\theta_1 q_H + q_H^2 - q_L}{2}. \tag{13}$$

Notice that this best reply obtains from the same reaction function, irrespectively of the extent of market coverage.[4] The profit functions simplify as follows:

$$\pi_H = \frac{(q_H^2 - \theta_1 q_H + q_L)^2}{4(q_H - q_L)}; \quad \pi_L = \frac{q_L(\theta_1 - 1 - q_L)}{2(q_H - q_L)}(2q_H - \theta_1 q_H + q_H^2 - q_L). \tag{14}$$

On the basis of (14), we are going to show the following

Proposition 2 For any $\theta_1 \in (1.6032, 9/4)$, there exists no subgame perfect equilibrium in pure strategies in the two-stage game.

Proof. Examine the first derivative of the high-quality firm's profit function w.r.t. q_H:

$$\frac{\partial \pi_H}{\partial q_H} = \frac{(q_L - \theta_1 q_H + q_H^2)(2\theta_1 q_L - q_L - \theta_1 q_H - 4q_H q_L + 3q_H^2)}{16(q_H - q_L)^2}, \tag{15}$$

[4] The reaction function of the high-quality firm is the same in both regimes because the definition of the demand function for the high-quality good is independent of the extent of market coverage.

which is equal to zero at $\hat{q}_H = \left(\theta_1 + \sqrt{\theta_1^2 - 4q_L}\right)/2$, where the profits of the high-quality firm are minimised and amount to $\pi_H = 0$ because $p_H = \hat{q}_H^2$ and $x_H = 0$. Notice that no level of q_H larger than $\hat{q}_H$ is acceptable, since it would imply both $p_H < q_H^2$ and $x_H < 0$.

Moreover, there exists a quality $q_H^M = \left(\theta_1 - 2 + \sqrt{(\theta_1 - 2)^2 + 4q_L}\right)/2$ such that $x_L = 0$, i.e., the high-quality firm becomes a monopolist. It can be shown that $q_L < q_H^M < \hat{q}_H$ for all $q_L < \theta_1 - 1$. The latter inequality is verified both in the right neighbourhood of $\theta_1 = 1.6032$, and in the left neighbourhood of $\theta_1 = 9/4$. A sufficient condition for this to obtain for any $\theta_1 \in (1.6032; 9/4)$, is that q_L is not increasing too fast as θ_1 increases. To verify this, consider that a viability condition for the low-quality firm, for any level of the marginal willingness to pay, is $\theta q_L \geq q_L^2$, given that price has to be at least equal to unit production cost. Any increase in product quality yields

$$\theta dq_L \geq 2q_L dq_L \tag{16}$$

which simplifies to $q_L \leq \theta/2$, implying that $\partial q_L/\partial\theta \leq 1/2$.[5]

The above discussion implies that, for any admissible level of q_L, the high-quality firm finds it profitable to produce q_H^M. If so, the low-quality firm must increase her quality-price ratio in order to have a positive demand. To this aim, she could either increase the quality or decrease the price. Obviously, the first option is not viable, in that for all q_L there exists a $q_H^M(q_L)$ such that the profits of the low-quality firm are nil. The second option, i.e., setting any price $p_L < (\theta_1 - 1)q_L$, contradicts Lemma 1.

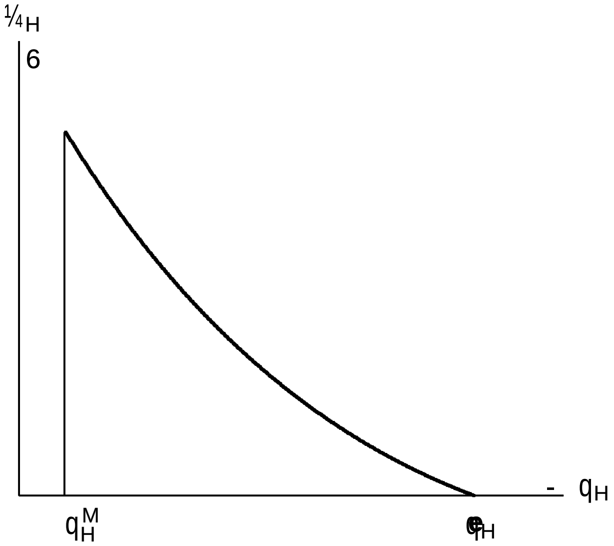
Alternatively, one can imagine that, when the high-quality firm is producing q_H^M, the low-quality firm could find it profitable to leapfrog the rival. In such a case, firms would exchange roles, and the former high-quality firm should set a price such that the surplus of the consumer located at $\theta_1 - 1$ would be nil. If so, the whole argument above repeats. ■

The intuition behind the above result can be outlined as follows. If the low-quality firm's price is such that the poorest consumer in the market enjoys zero surplus, then the high-quality firm finds it profitable to decrease her quality level to q_H^M, in order to increase her market share. The reaction of the low-quality firm would be to allow the consumer in θ_0 to enjoy a positive surplus, either by decreasing price, which violates the market demand

[5] This result has been shown by Delbono, Denicolò and Scarpa (1996).

specification, or by offering a higher quality. However, since q_H^M exists for all acceptable levels of q_L; increasing quality does not yield positive profits for the low-quality firm. The combination of these two facts prevents firms from reaching a pure-strategy equilibrium in prices. The shape of the high-quality firm's profit function is illustrated in Figure 1.

Figure 1 : The high-quality firm's profit function



Finally, we briefly discuss the above results in contrast with the analysis conducted by Wauthy (1996) in a model where firms produce at no cost. Under this assumption, any incentive for the high-quality producer to decrease quality in order to reduce production costs, and/or increase the market share for her variety, disappears and her profit function is everywhere increasing in q_H: Hence, a pure-strategy subgame perfect equilibrium in qualities and prices always exists, under either full or partial market coverage, or when a corner solution arises at the price stage (see Proposition 1 in Wauthy (1996, p. 348)).

4 Concluding remarks

In the foregoing analysis, we have investigated the existence of a pure-strategy subgame perfect equilibrium in a duopoly model of vertical differentiation with convex variable costs of quality. We have shown that there are parameter ranges where a pure strategy equilibrium exists (i) under partial market coverage, if consumers' marginal willingness to pay for quality is relatively low; (ii) under full market coverage, if consumers' marginal willingness to pay for quality is relatively high. However, these two intervals are disjoint. In such intermediate parameter range, we have proved that the low-quality firm is constrained to price so as to extract all the surplus from the poorest consumer in the market. This, in turn, induces the high-quality firm to decrease her quality towards the rival's, in order to increase her market share. This argument, in combination with the possibility for the low-quality firm to leapfrog the rival, entails that a pure-strategy duopoly equilibrium does not exist. This cannot happen in a model where production costs are nil, as assumed by Wauthy (1996).

The above findings reveal that, contrary to previous beliefs, vertical differentiation models suffer from a problem of non-existence of the equilibrium in pure strategies which affects spatial differentiation models. While in spatial models the non-existence is due to an insufficient degree of convexity of transportation costs, in vertical differentiation models it appears to be due to the convexity of production costs in a subset of the parameter space where a corner solution in prices is the unique candidate as a Nash equilibrium at the market stage.

References

[1] Anderson, S.P. (1988), ”Equilibrium Existence in the Linear Model of Spatial Competition”, Economica, 55, 479-91.

[2] Anderson, S.P., A. de Palma and J.-F. Thisse (1992), Discrete Choice Theory of Product Di¤erentiation, Cambridge, MA, MIT Press.

[3] Champsaur, P. and J.-C. Rochet (1989), ”Multiproduct Duopolists”, Econometrica, 57, 533-57.

[4] Choi, J.C. and H.S. Shin (1992), ”A Comment on a Model of Vertical Product Di¤erentiation”, Journal of Industrial Economics, 40, 229-31.

[5] Cremer, H. and J.-F. Thisse (1991), ”Location Models of Horizontal Differentiation: A Special Case of Vertical Di¤erentiation Models”, Journal of Industrial Economics, 39, 383-90.

[6] Cremer, H. and J.-F. Thisse (1994), ”Commodity Taxation in a Di¤erentiated Oligopoly”, International Economic Review, 35, 613-33.

[7] Dasgupta, P. and E. Maskin (1986), ”The Existence of Equilibrium in Discontinuous Economic Games, II: Applications”, Review of Economic Studies, 53, 27-42.

[8] d’Aspremont, C., J.J. Gabszewicz and J.-F. Thisse (1979), ”On Hotelling’s ‘Stability in Competition”’, Econometrica, 47, 1145-50.

[9] Delbono, F., V. Denicolò and C. Scarpa (1996), ”Quality Choice in a Vertically Di¤erentiated Mixed Duopoly”, Economic Notes, 25, 33-46.

[10] Economides, N. (1986), ”Minimal and Maximal Di¤erentiation in Hotelling’s Duopoly”, Economics Letters, 21, 67-71.

[11] Gabszewicz, J.J. and J.-F. Thisse (1979), ”Price Competition, Quality and Income Disparities”, Journal of Economic Theory, 20, 340-59.

[12] Gabszewicz, J.J. and J.-F. Thisse (1980), ”Entry (and Exit) in a Di¤erentiated Industry”, Journal of Economic Theory, 22, 327-38.

[13] Gabszewicz, J.J. and J.-F. Thisse (1986), "On the Nature of Competition with Di¤erentiated Products", Economic Journal, 96, 160-72.

[14] Hotelling, H. (1929), "Stability in Competition", Economic Journal, 39, 41-57.

[15] Lambertini, L. (1996), "Choosing Roles in a Duopoly for Endogenously Di¤erentiated Products", Australian Economic Papers, 35, 205-24.

[16] Moorthy, K.S. (1988), "Product and Price Competition in a Duopoly Model", Marketing Science, 7, 141-68.

[17] Motta, M. (1993), "Endogenous Quality Choice: Price vs Quantity Competition", Journal of Industrial Economics, 41, 113-32.

[18] Osborne, M.J. and C. Pitchick (1987), "Equilibrium in Hotelling's Model of Spatial Competition", Econometrica, 55, 911-22.

[19] Shaked, A. and J. Sutton (1982), "Relaxing Price Competition through Product Di¤erentiation", Review of Economic Studies, 69, 3-13.

[20] Shaked, A. and J. Sutton (1983), "Natural Oligopolies", Econometrica, 51, 1469-83.

[21] Tirole, J. (1988), The Theory of Industrial Organization, Cambridge, MA, MIT Press.

[22] Wauthy, X. (1996), "Quality Choice in Models of Vertical Di¤erentiation", Journal of Industrial Economics, 44, 345-53.